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BREMSSTRAHLUNG CONVERTER CONSIDERATIONS

P. Axel

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Physics Research Laboratory
Physics Department
University of Illinois
Urbana, Illinois

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I. INTRODUCTION

The choice of the proper converter (i.e., bremsstrahlung target) thickness and the proper converter material depends on both the experiment being performed and on the backgrounds. Any choice will be a compromise between the factors listed below. In many cases, some relatively obvious alternatives will have both positive and negative features. It is quite likely that no decisive feature will emerge, and that one alternative will be chosen arbitrarily in preference to spending the excessive times probably necessary to find the optimum condition experimentally.

A. Low Z vs. High Z; Collision Loss

One of the easiest choices to define is that between low Z and high Z target materials. To a first approximation, the ratio of bremsstrahlung to multiple scattering is independent of Z. If the usable targets are thick enough for electron energy loss variation to be appreciable, high Z targets are to be preferred. (This is probably the case for targets as thick as 0.002 radiation lengths.) It is conceivable, although unlikely, that photonuclear events in the converter could produce annoying background; if this were true a material with a high photonuclear threshold (and therefore probably a low Z) would be better. On the other hand if secondary Compton scattering in the converter were a factor, high Z materials would be better.

It should be noted that the <u>mean</u> electron energy loss due to collision is not a source of error because the <u>mean</u> energy loss is relatively independent of energy. Thus, the primary electron and the post-bremsstrahlung electron together lose a constant mean energy independent of where in the converter the photon originates. However, energy loss can introduce an energy uncertainty because there is a spread of about 20% about the mean energy loss and because some electrons lose considerably greater energies. A high Z material is better because the radiation intensity is proportional to Z^2 whereas the energy loss (which is caused by collisions with electrons) is proportional to Z.

B. Target Thickness and Background

The choice of target thickness is much more complex because it is based on a compromise between the background, the resolution, and the counting rate.

Consider first five sources of background.

<u>Type 1</u>. The majority of incident electrons pass through the converter without interacting and enter the sample area. The position at which these leave the spectrometer, and the direction in which they go depend on the spectrometer magnet setting. If the spectrometer field is larger, this background will probably be smaller; on the other hand, for a given gamma ray energy, a larger spectrometer field implies a higher energy incident electron beam which might produce more background. This background can be reduced by reducing the incident electron beam (and compensating by using a thicker target).

<u>Type 2.</u> The converter background from non-bremsstrahlung events (mentioned above in the discussion of low Z vs. high Z) is probably negligible.

<u>Type 3.</u> For sufficiently thin converters, the non-monochromatic photons (i.e., those corresponding to electron energies not being accepted by the electron detector) produce background events when they interact with the sample. (The sample would give the main effect if the true events being detected originate in the sample.) For a given incident electron energy, the ratio of true events to this background cannot be changed (if the converter is already thin). In many cases however, using relatively low incident energies will reduce this background by reducing the unused high energy photons.

<u>Type 4.</u> For thicker converters, some of the electrons may be multiply scattered to such an extent that they miss the electron detectors. In this case, background of type 3 increases relative to background of type 1. (In some rare resonance reactions, it is conceivable that the photons of the correct energy, whose electrons have not reached the electron detectors, would contribute anomalously to "chance" events.) This type of background enhancement can be decreased by selecting higher energy post bremsstrahlung electrons.

Type 5. Low energy post-bremsstrahlung electrons may cause background by hitting the spectrometer after having suffered excessive multiple scattering. This background is probably negligible inasmuch as its energy would be very low. It can be influenced only by changing the target thickness.

II. RESOLUTION

The resolution enters the choice of converter in two ways. First, if type 4 background (due to multiple scattering) is serious, it could be reduced by detecting a higher energy post-bremsstrahlung electron in the spectrometer. This deviation from perfect dispersion matching (of dispersion magnet and spectrometer magnet) will impair the resolution. Second, if there should be any advantage to using a low Z material (due to background type 2), the spread in energy losses due to collision would hurt the resolution.

Note that according to the graph on p. 132 of Technical Report #21 (Bremsstrahlung Monochromator Report), the loss in resolution due to large multiple scattering angles is probably negligible. For a multiple scattering angle as large as 8°, the image shifts only about 2 mm at the extremes of the spectrometer and much less than this at the center of the spectrometer. (The 2 mm assumes $\Delta z/z$ of about 3×10^{-3}, z about 60 cm, and K between 0.55 and 0.9 for the parameters on p. 132.) (Note: the typed z is the same as the script z used in the report.)

There are three principal causes of energy broadening:

1. Size of Monochromatic Focus at Converter

If monoenergetic electrons form an image with a horizontal length of L_1 cm (along the z axis of the spectrometer), an equivalent energy spread ΔE_1 (in kev) will be produced. Define the dispersion, d_s, of the

spectrometer magnet as:

$$d_s = \frac{\Delta E_s}{\Delta L} \text{ in Kev/cm} \tag{1}$$

For the spectrometer

$$\frac{\Delta E_s}{E_e} \frac{1}{\Delta L} = 1.18\% \text{ / cm} \tag{2}$$

If ΔE is in Kev, E in Mev, and ΔL in cm:

$$d_s = 11.8 \, E_e \tag{3}$$

d_s varies from this by $\pm$ 12% at different exit positions on the spectrometer. d_s is largest when the distance between converter and detector (along the z axis) is least.

2. Finite Size of Detector

If the electron detector has a length L_2 cm (along the z axis), the energy range accepted, ΔE_2, is:

$$\Delta E_2 = L_2 \, (11.8) \, E_e \tag{4}$$

3. Energy Spread in the Incoming Beam and Dispersion Mismatch

The energy of the electrons from the betatron, E_β, may vary. This variation may be due to poor regulation or due to the finite pulse length. Insofar as the energy spread is due to poor regulation, auxiliary timing and pulse height circuits can be used to eliminate the counts for the wrong E_β. Of course, this has the disadvantage of reducing the usable intensity.

The variation in E_β due to finite pulse length could also be taken care of, in principle, with timing circuits which could route pulses to different groups of addresses in the multichannel analyzer. Alternatively, the magnetic field of the dispersion magnet or the spectrometer magnet might be given a compensating 180 cycle component. However, these techniques would not help variations in E_β due to improper regulation.

The simplest way to overcome errors due to variations of E_β is to properly match the dispersions of the D-magnet (dispersion magnet) and of the S-magnet spectrometer magnet. To illustrate this, consider ΔE_β as the energy variation (in kev) due to the finite pulse length.
(E_β in Mev)

$$\Delta E_\beta = 10^3 \; E_\beta \left[1 - \mathrm{Sin}\left(\frac{\pi}{2} \pm \frac{\pi T}{5555}\right)\right] \tag{5}$$

where T is the pulse length in microseconds and the pulse is assumed centered at 90°.

Define:

$$p = \frac{\pi T}{5555} \tag{6}$$

For small T or p, Eq. (5) can be rewritten as:

$$\Delta E_\beta = 10^3 \; E_\beta \frac{p^2}{2} \left(1 - \frac{p^2}{12}\right) \tag{7}$$

$$\Delta E_\beta \simeq 10^3 \; E_\beta \frac{p^2}{2} \tag{7a}$$

Note that ΔE_β as given by Eq. (7a) is the

maximum variation of E_β. The mean value of the variation of E_β is only $(\Delta E_\beta / 3)$. The distribution of E_β as a function of t for a given pulse length T_o, can be obtained by substituting $E_{\beta o}$ for E_β and t for T in Eqs. (5-7).

Eq. (7a) can be rewritten with the aid of Eq. (6) as:

$$\Delta E_\beta = 1.61\ (T/100\ \mu sec)^2\ E_\beta \tag{8}$$

The length of the spot across the converter is:

$$L_c = (\Delta E_\beta / d_D) \tag{9a}$$

$$L_c = (\Delta E_\beta / 2.4\ E_\beta) \tag{9b}$$

where the dispersion of the D magnet in kev/cm is:

$$d_D = \frac{10^3}{E_\beta}\frac{\Delta E_\beta}{L_c}\ E_\beta \tag{10a}$$

$$d_D = 2.4\ E_\beta \tag{10b}$$

if $(10^3\ \Delta E_\beta / E_\beta\ L_c\ ) = 2.4$ corresponding to 0.24% / cm on the dispersion magnet.

In terms of T,

$$L_c = 0.67\ (T / 100\ \mu\ sec)^2 \tag{11}$$

The compensating energy shift, ΔE_e, of the post bremsstrahlung electrons, and the displacement, L_s, in z of the S magnet are:

$$\Delta E_e = L_c d_s = L_c (11.8) E_e \tag{12}$$

$$\Delta E_e = \Delta E_\beta (11.8 E_e / 2.4 E_\beta) \tag{13a}$$

$$\Delta E_e = \Delta E_\beta (4.92 E_e / E_\beta) \tag{13b}$$

$$\Delta E_e = 7.9 E_e (T / 100 \mu sec)^2 \tag{13c}$$

$$L_s = (\Delta E_\beta / d_s) = (\Delta E_\beta / 11.8 E_e) \tag{14}$$

$$L_s = 0.136 (E_\beta / E_e) (T / 100 \mu sec)^2 \tag{15}$$

The net effects, due to both dispersions are:

$$\Delta E = \Delta E_e - \Delta E_\beta \tag{16a}$$

$$\Delta E = (\Delta E_\beta / E_\beta) (4.92 E_e - E_\beta) \tag{16b}$$

If ΔE_β is due only to T,

$$\Delta E = 1.61 (T / 100 \mu sec)^2 (4.92 E_e - E_\beta) \tag{17}$$

Similarly:

$$\Delta L = L_s - L_c \tag{18}$$

$$\Delta L = \Delta E_\beta \left[\frac{1}{11.8 E_e} - \frac{1}{2.4 E_\beta} \right] \tag{19a}$$

$$\Delta L = (\Delta E_\beta / E_\beta) \left[(E_\beta - 4.9 E_e) / 11.8 E_e \right] \tag{19b}$$

If ΔE_β is due only to T,

$$\Delta L = (E_\beta - 4.9 E_e) (0.136 / E_e) (T/ 100 \mu sec)^2 \tag{20}$$

Table I gives some typical values for the parameters involved in dispersion matching.

Table I

Energy Spreads and Resolution Data Due to Long Pulse

	Pulse Length µsec					
	50	100	150	200	250	300
$10^3(1-\sin\theta) = 10^3 p^2/2$	0.40	1.61	3.62	6.43	10.0	14.4
Length on z axis of perfectly focusses spot (in cm) if D magnet 0.24% / cm	0.17	0.67	1.50	2.68	4.17	6.00

ΔE_β = Energy Spread in Kev at Bremsstrahlung target (kev)

Peak Energy (Mev)	d_D Kev/cm	Pulse Length in µsec 50	100	150	200	250	300
24	58	9.6	38	86	154	240	346
22	53	8.8	35	79	141	220	317
20	48	8.0	32	72	128	200	288
18	43	7.2	29	65	116	180	260
16	38	6.4	26	58	103	160	230
14	34	5.6	22	50	90	140	220
12	29	4.8	19	43	77	120	173
10	24	4.0	16	36	64	100	144

ΔE_e = Energy Compensation at e^- Counter (kev)

(Using 1.18% for S magnet; ±0.14% variation over S magnet)

E_e^-	4.92 E_e^-	d_s Kev/cm						
2	9.8	24	3.9	16	36	63	98	142
4	19.7	47	7.9	32	71	126	197	284
6	29.5	70	11.8	47	107	190	295	425
8	39.4	94	15.7	63	142	253	394	577
10	49.2	118	19.7	79	178	316	492	709

III. RADIATION INTENSITY AND MULTIPLE SCATTERING

The calculation of the radiation intensity and the multiple scattering for a given target are illustrated below. Typical values are given for four thicknesses of Al and for four different thicknesses of Pt (which are chosen to give the same radiation intensity). For each converter, the energy loss is listed and the multiple scattering is calculated for three energies of post-bremsstrahlung electrons. (The multiple scattering calculations show that a high Z material has slightly less multiple scattering than a low Z material.)

A. Radiation Intensity

The exact bremsstrahlung cross section is quite complex and has not yet been calculated completely.[1,2,3] However, the simplest theory gives results adequate for target considerations.

One defines an effective total radiation cross section, Φ_{rad}, which is:

$$\Phi_{rad} = \frac{1}{N} \frac{1}{E_o} \left(\frac{dE_o}{dx}\right)_{rad} \tag{21}$$

where N = atoms/cm^3, $(dE_o/dx)_{rad}$ is the energy lost in radiation per cm, and E_o is the incident energy.

A radiation length, X_o, is defined as the number of grams/cm^2 of target necessary for an electron to radiate

(on the average) its own energy in the target. For low and medium energies, X_o is a function of E_o. However, for high energy (due to screening effects), X_o is independent of E_o. If Φ_{rad} is known, X_o can be found easily from: :

$$X_o = \frac{A}{N_o \, \Phi_{rad}} \tag{22}$$

where A is the atomic weight and N_o is Avagadro's number. It is often convenient to get the thickness in cm, L, which would cause an energy loss equal to E_o, the incident energy:

$$L = \frac{X_o}{\rho} \tag{23}$$

where ρ is the density.

If Q electrons of energy E_o are incident on a target of thickness t, the number of photons of energy E_γ in an interval ΔE is given approximately by:

$$N(E_\gamma)\ \Delta E = Q\ \frac{t}{X_o}\ \frac{\Delta E}{E_\gamma} \tag{24}$$

Eq. (24) involves the approximation that $E_\gamma N(E_\gamma)$ is a constant; it overestimates the number of high energy photons and underestimates the number of low energy photons (by about 20%). More accurate approximations than Eq. (24) become quite complicated because they involve screening and coulomb corrections to the plane wave normally used for the electron. (Fortunately graphs are available[2] of $N(E_\gamma)$ dE.)

X_o can be determined easily if screening and coulomb corrections are neglected:

$$\Phi_{rad} = \bar{\Phi} \left(4 \log \frac{2E_o}{mc^2} - \frac{4}{3}\right) \tag{25}$$

where

$$\bar{\Phi} = Z(Z + a) \quad 5.79 \times 10^{-28} \text{ cm}^2 \tag{26}$$

and a = 1 to take into account radiation due to electron-electron bremsstrahlung. The neglect of screening in Eq. (24) and Eq. (26) is valid in the limit $mc^2 \ll E_o \ll 137\ mc^2\ Z^{-1/3}$. The bracket in Eq. (25) would have the values 17.1, 16.2, 15.0, 13.4, and 10.6 for 25, 20, 15, 10, and 5 Mev electrons.

In the limit of complete screening, $E_o \gg 137\ mc^2\ Z^{-1/3}$, Eq. (25) is replaced by

$$\Phi_{rad} = \bar{\Phi} \left(4 \log 183\ Z^{-1/3} + \frac{2}{9}\right) \tag{27}$$

where $\bar{\Phi}$ is given by Eq. (26) except that a varies slightly with Z; a is 1.28 for Al and 1.17 for Pt. The bracket in Eq. (27) has the values 14.6 and 17.0 for Al and Pt.

Following Koch and Motz[2] we shall use $\Phi_{rad} / \bar{\Phi} = 14$ and 12 for Al and Pt at 20 Mev. These numbers would not change much at nearby energies; the corresponding values at 5 Mev would be about 11.0 and 10.5.

These values give $\Phi_{rad} = 1.48 \times 10^{-24}$ cm^2 and 4.28×10^{-23} cm^2 for Al and Pt at 20 Mev. The corresponding

radiation lengths are 1.12 and 3.95 x 10^{-2} moles/cm^2 or 30.3 gm/cm^2 and 7.7 gm/cm^2.

Table II gives the characteristics of the targets to be considered. The energy loss which is almost independent of energy is obtained from the graphs given by Hansen and Fultz.[4]

Table II

Target Characteristics

Material	Rad. Lengths	0.001	0.002	0.005	0.01
Al	mg/cm^2	30.3	60.6	152	303
	10^{-3}moles/cm^2	1.21	2.42	6.05	12.1
	10^{-2} cm	1.12	2.24	5.6	11.2
	mils	4.59	9.18	22.9	45.9
	Energy Loss (kev)[a] 4.2 Mev/cm	47	98	235	471
Pt	mg/cm^2	7.7	14.4	38.5	77
	10^{-5}moles/cm^2	3.95	7.90	19.7	39.5
	10^{-3} cm	0.36	0.72	1.81	3.62
	mils	0.147	0.295	0.74	1.47
	Energy Loss (kev)^2 22 Mev/cm	7.9	15.8	39.5	79

(a) The spread in energy loss is about 20% of the listed value; there is also a group of electrons which lose significantly more energy

B. Multiple Scattering

1. Approximate Treatment

Even though multiple electron scattering is quite complicated, its main features can be summarized simply. If electrons all traverse the same thickness of material, to a good approximation, the probability of finding an emerging electron that has been scattered between θ and $\theta + d\theta$ is given by:

$$P_1(\theta)\, d\theta = \frac{2\theta d\theta}{\theta_w^2}\; e^{-\theta^2/\theta_w^2} \qquad (28)$$

Eq. (28) is the form obtained from approximate theory and has been shown to be a very good approximation to the more exact theory if θ_w is reinterpreted.[5]

For a given number of atoms/cm^2, θ_w is proportional to Z/E where E is the electron energy. θ_w is also proportional to the square root of the foil thickness or to the number of atoms/cm^2. If the foil thickness, t, is expressed in grams/cm^2, θ_w^2 is proportional to Z^2t/AE^2.

For the targets being considered, an approximate form of θ_w is:

$$\text{For Al}\quad \theta_w = 11.5^\circ \left(\frac{\text{Thickness}}{60.6\ \text{mg/cm}^2}\right)^{1/2} \left(\frac{3\ \text{Mev}}{E_e}\right) \qquad (29)$$

$$\text{For Pt}\quad \theta_w = 11.0^\circ \left(\frac{\text{Thickness}}{15.4\ \text{mg/cm}^2}\right)^{1/2} \left(\frac{3\ \text{Mev}}{E_e}\right) \qquad (30)$$

where E_e is the energy of the post bremsstrahlung electrons, and the thicknesses that appear in the denominator correspond to 0.002 radiation lengths. More exact calculations change the exponents in Eqs. (29) and (30) to $t^{0.59}$ and $E_e^{-0.87}$. Using Eqs. (29) and (30) can underestimate θ_w by about 15% for large thicknesses.

2. More Exact Evaluation of θ_w

In order to calculate more exact values of θ_w, auxiliary parameters are introduced. The most important of these is θ_1 which contains the principal Z, E_e, and t dependence of θ_w.

$$\theta_1^2 = 0.175 \frac{Z(Z+1)}{A} \frac{t}{(pv)^2} \tag{31}$$

where (pv) is in Mev. If $E_e' = E_e + 0.51$ is the total electron energy (including rest mass) in Mev,

$$(pv)^2 = \beta^2 (E_e'^2 - 0.26) \tag{32}$$

Thus $(pv)^2$ = 5.79, 11.8, and 19.7 for 2, 3, and 4 Mev electrons.

For the range of target thicknesses under discussion θ_w / θ_1 varies slowly from 2.52 to 2.99 for Al and from 2.07 to 2.65 for Pt. (From Eq. (31) it is clear that to obtain the same θ_1 from two materials the product of the number of atoms per cm^2 and Z(Z + 1) should be kept constant.)

The values of θ_1 for 100 mg/cm^2 foils are given in table III:

Table III

θ_1 (radians)

	2 Mev	3 Mev	4 Mev
Al	0.135	0.096	0.074
Pt	0.309	0.217	0.167

The ratio, θ_w / θ_1, depends on the screening correction which in turn depends mainly on the target thickness t. The screening correction is calculated by finding first the parameter:

$$\left(\theta_1 / \theta_a\right)^2 = 7800 \, \frac{(Z + 1)\, Z^{1/3}}{\beta^2 \, A(1 + 3.35\alpha^2)} \, t \qquad (33)$$

where $\alpha = \frac{Ze^2}{\hbar v}$. Note that θ_a is an effective minimum scattering angle from single scattering theory; it is not needed explicitly for multiple scattering calculations because only the ratio given in Eq. (33) enters.

There is only a very slight energy dependence in Eq. (33); it enters only in the form of $\beta = \frac{v}{c}$. For example, $(\theta_1 / \theta_a)^2$ has the values (for 2 Mev, 3 Mev, and 4 Mev electrons) of 9600t, 9430t and 9350t for Al and of 6610t, 6580t, and 6550t for Pt.

The screening correction depends only logarithmically on $(\theta_1 / \theta_a)^2$. The screening correction depends on b, which is:

$$b = \ln (\theta_1 / \theta_a)^2 - 0.15 \quad (34)$$

The parameter, b, in turn determines B:

$$B - \ln B = b \quad (35)$$

Finally, θ_w / θ_1 is:

$$\frac{\theta_w}{\theta_1} = (B - 1.2)^{1/2} \quad (36)$$

Table IV lists θ_w as well as the parameters defined above which were used in its calculation. The final eight lines in Table IV are discussed below.

Table IV

Radiation Lengths	0.001	0.002	0.005	0.01
Aluminum				
$10^3 t$ (mg/cm^2)	30.3	60.6	152	303
$(t/0.1\ \mathrm{gm/cm^2})^{1/2}$	0.55	0.78	1.23	1.74
$(\theta_1/\theta_a)^2$	287	574	1440	2870
$b = \ln\left[(\theta_1 / \theta_a)^2\right] - 0.15$	5.51	6.20	7.12	7.80
B	7.55	8.34	9.38	10.13
$\theta_w / \theta_1 = (B - 1.2)^{1/2}$	2.52	2.68	2.86	2.99

Radiation Lengths	0.001			0.002			0.005			0.01		
Elect. Kinetic Energy (Mev)	2	3	4	2	3	4	2	3	4	2	3	4
θ_1 (deg)	4.27	3.02	2.34	6.04	4.28	3.3	9.54	6.77	5.22	13.5	9.56	7.38
θ_w (deg)	10.7	7.6	5.9	16.2	11.5	8.85	27.3	19.4	14.9	40.3	28.5	22.1

Table IV (Cont'd.)

Radiation Lengths	0.001			0.002			0.005			0.01		
<u>Pt</u>												
$10^3 t$ (mg/cm^2)	7.7			15.4			38.5			77		
$(t/0.1\ \mathrm{gm/cm^2})^{1/2}$	0.28			0.39			0.62			0.88		
$(\theta_1 / \theta_a)^2$	50.7			101			254			507		
b	3.79			4.46			5.39			6.10		
B	5.5			6.3			7.4			8.23		
θ_w / θ_1	2.07			2.26			2.49			2.65		
Elect. En. (Mev)	2	3	4	2	3	4	2	3	4	2	3	4
θ_1 (deg)	4.92	3.45	2.66	6.94	4.88	3.76	11.0	7.7	5.94	15.5	10.9	8.4
θ_w (deg)	10.2	7.13	5.51	15.7	11.0	8.52	27.4	19.2	14.8	41.0	28.3	22.2
% e^- within												
± 5°	48	66	80	27	44	57	13	21	30	6	12	18
± 8°	73	90	96	50	68	82	24	39	53	13	23	34
± 10°	85	95	97	62	80	92	32	51	64	19	30	46
± 15°	97	99	99	83	96	98	53	72	86	32	51	64
Relative Yield in												
± 5°	29	40	49	33	54	69	39	64	91	39	73	109
± 8°	45	55	59	61	83	100	73	118	161	79	140	207
± 10°	53	58	59	76	98	112	97	155	195	115	183	280
± 15°	59	60	60	102	117	120	161	219	262	195	310	390

3. Bremsstrahlung Production Throughout Converter

The values of θ_w in table Iv could be substituted in Eq. (28) to give the angular distribution of electrons if the post-bremsstrahlung electron always traversed the entire target thickness. However, the post-bremsstrahlung electrons are formed uniformly throughout the target. This uniform production can be treated analytically by making two approximations. First, neglect the multiple scattering of the incident electron. (Since θ_w is approximately proportional to E^{-1} or $E^{-0.87}$, this will introduce relatively little error if the post-bremsstrahlung electron has much lower energy than the incident electron). The second approximation assumes that the θ_t appropriate to a residual thickness t can be written as:

$$\theta_t^2 = \frac{t}{t_m} \theta_w^2 \qquad (37)$$

where t_m is the maximum thickness the electron can go through (i.e., t_m is the entire target thickness for which θ_w is the corect value). This approximation assumes that the angular distribution is always Gaussian and that the 1/e value is proportional to $t^{1/2}$ (rather than $t^{0.59}$ suggested above).

With these approximations the probability of scattering into an angle θ can be written as:

$$P(\theta)d\theta = \frac{1}{t_m} \int_0^{t_m} dt \ \frac{2\theta}{\theta_t^2} \ e^{-\theta^2/\theta_t^2} \ d\theta \tag{38}$$

$$= \frac{2\theta d\theta}{\theta_w^2} \int_0^{t_m} \frac{dt}{t} \ e^{-\theta^2 t_m/\theta_w^2 t} \tag{39}$$

$$P(\theta)d\theta = \frac{2\theta d\theta}{\theta_w^2} \left[-E_i \ \left(- \frac{\theta^2}{\theta_w^2}\right) \right] \tag{40}$$

The angular distribution of electrons leaving the foil, $P(\theta)$, as given by Eq. (41), is much more concentrated at small angles than a Gaussian (i.e., $P_1(\theta)$ from Eq. (28)) would be.

For the monochromator, we are interested in the fraction of electrons which have been scattered by less than some angle, say θ_m. This fraction $F(\theta_m)$ is given by:

$$F(\theta_m) = \int_0^{\theta_m} P(\theta) \ d\theta \tag{41}$$

This integral can be evaluated easily; it is convenient to introduce the notation:

$$y = (\theta/\theta_w)^2 \text{ and } y_m = (\theta_m/\theta_w)^2 \tag{42}$$

From Eq. (40-42), we then get:

$$F(\theta_m) = \int_0^{y_m} dy \ \left[-E_i(-y)\right] \tag{43}$$

$$= y_m \left[-E_i \ (-y_m)\right] + 1 - e^{-y_m} \tag{44}$$

Table V gives the values of $F(\theta_m)$ as a function of y_m.

Table V

$(\theta_m/\theta_w)^2$ =	0.01	0.04	0.1	0.2	0.3	0.4	0.5
$F(\theta_m)$ =	0.05	0.146	0.275	0.437	0.531	0.61	0.673
$(\theta_m/\theta_w)^2$ =	0.6	0.7	0.8	0.9	1.0	1.2	1.4
$F(\theta_m)$ =	0.72	0.76	0.80	0.83	0.85	0.89	0.92
$(\theta_m/\theta_w)^2$ =	1.6	1.8	2.0	2.5	3.0	4.0	5.0
$F(\theta_m)$ =	0.94	0.95	0.96	0.97	0.98	0.985	0.99

The results of this analysis for the Pt target are included in the last eight lines of table IV. The first four (of these last eight) lines give the percentage of post bremsstrahlung electrons of the indicated energy which have been scattered by ± 5°, ± 8°, ± 10°, and ± 15°. The vertical opening on the spectrometer magnet is ± 8°.

The final four lines give the relative yields of electrons within these angles if the same incident electron beam is used for all four target thicknesses. The examples below illustrate the implications of these relative yields.

Example A

Let us compare the 0.002 and 0.005 radiation length targets for 4 Mev electrons. Assume first that an 8° cone is acceptable. In order to obtain the same number of usable photons, the thicker target requires (100/161) = 0.62 of the beam. Therefore, both the high energy electron beam

leaving the spectrometer and the background produced by this beam decrease by 0.62. On the other hand, the total number of photons produced and the total number of post-bremsstrahlung electrons of degraded energy is (0.005/0.002) (100/161) = 1.55 times as high. Thus, if the background or counting rate limitation is due to events in the target or sample (as opposed to being due to the main beam), the thicker target is inferior.

Example B

Let us compare getting 8 Mev gamma rays from a combination of incident emergent electron energies of 10 Mev - 2 Mev or 12 Mev - 4 Mev. Consider a 0.002 radiation length converter. 10^{-9} amp of 10 Mev electrons would give the same number of usable photons as would 6.1 (10^{-10}) amps of 12 Mev electrons. To simplify comparison, assume the same peak current but have the 12 Mev current pulse only 180 μsec long while the 10 Mev current is 300 μsec long. At the converter, the energy spread of the incident beam would be 66 kev (for the 12 Mev) and 144 kev. After compensation, the energy spreads of the 8 Mev gamma ray would be 34 kev (for 4 Mev electrons) and 2 kev. The electron beam currents entering the sample area would be the same (per unit of "on" time) but the higher energy 12 Mev electrons would produce more background.

(This background might be less intense at the detector because the deflection of 12 Mev electrons when the spectrometer is set at 4 Mev would be greater than the deflection of 10 Mev electrons when the spectrometer is set at 2 Mev.) On the other hand, the actual counting rate per unit "on" time would be smaller for the 2 Mev electrons. (Note that 50% of the radiation produced by the 10 Mev electrons is not usable, if 8° is the actual limitation, whereas only 18% of the radiation produced by 12 Mev electrons is not usable.

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   p. 252-259 for collision energy loss
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Table I

Energy Spreads and Resolution Data Due to Long Pulse

	Pulse Length μsec					
	50	100	150	200	250	300
$10^3(1-\sin\theta) = 10^3 p^2/2$	0.40	1.61	3.62	6.43	10.0	14.4
Length on z axis of perfectly focusses spot (in cm) if D magnet 0.24% / cm	0.17	0.67	1.50	2.68	4.17	6.00

ΔE_β = Energy Spread in Kev at Bremsstrahlung target (kev)

Peak Energy (Mev)	d_D $\frac{\text{Kev}}{\text{cm}}$	Pulse Length in μsec: 50	100	150	200	250	300
24	58	9.6	38	86	154	240	346
22	53	8.8	35	79	141	220	317
20	48	8.0	32	72	128	200	288
18	43	7.2	29	65	116	180	260
16	38	6.4	26	58	103	160	230
14	34	5.6	22	50	90	140	220
12	29	4.8	19	43	77	120	173
10	24	4.0	16	36	64	100	144

ΔE_e = Energy Compensation at e^- Counter (kev)

(Using 1.18% for S magnet; ±0.14% variation over S magnet)

E_e^-	$4.92\ E_e^-$	d_s $\frac{\text{Kev}}{\text{cm}}$						
2	9.8	24	3.9	16	36	63	98	142
4	19.7	47	7.9	32	71	126	197	284
6	29.5	70	11.8	47	107	190	295	425
8	39.4	94	15.7	63	142	253	394	577
10	49.2	118	19.7	79	178	316	492	709

Table II

Target Characteristics

Material	Rad. Lengths	0.001	0.002	0.005	0.01
Al	mg/cm^2	30.3	60.6	152	303
	10^{-3} $moles/cm^2$	1.21	2.42	6.05	12.1
	10^{-2} cm	1.12	2.24	5.6	11.2
	mils	4.59	9.18	22.9	45.9
	Energy Loss (kev)[a] 4.2 Mev/cm	47	98	235	471
Pt	mg/cm^2	7.7	14.4	38.5	77
	10^{-5} $moles/cm^2$	3.95	7.90	19.7	39.5
	10^{-3} cm	0.36	0.72	1.81	3.62
	mils	0.147	0.295	0.74	1.47
	Energy Loss (kev)[2] 22 Mev/cm	7.9	15.8	39.5	79

(a) The spread in energy loss is about 20% of the listed value; there is also a group of electrons which lose significantly more energy

Table IV

Radiation Lengths	0.001	0.002	0.005	0.01
<u>Aluminum</u>				
$10^3 t$ (mg/cm^2)	30.3	60.6	152	303
$(t/0.1\ \mathrm{gm/cm^2})^{1/2}$	0.55	0.78	1.23	1.74
$(\theta_1/\theta_a)^2$	287	574	1440	2870
$b = \ln\left[(\theta_1 / \theta_a)^2\right] - 0.15$	5.51	6.20	7.12	7.80
B	7.55	8.34	9.38	10.13
$\theta_w / \theta_1 = (B - 1.2)^{1/2}$	2.52	2.68	2.86	2.99

Radiation Lengths	0.001			0.002			0.005			0.01		
Elect. Kinetic Energy (Mev)	2	3	4	2	3	4	2	3	4	2	3	4
θ_1 (deg)	4.27	3.02	2.34	6.04	4.28	3.3	9.54	6.77	5.22	13.5	9.56	7.38
θ_w (deg)	10.7	7.6	5.9	16.2	11.5	8.85	27.3	19.4	14.9	40.3	28.5	22.1

Table IV (Cont'd.)

Radiation Lengths	0.001			0.002			0.005			0.01		
<u>Pt</u>												
$10^3 t$ (mg/cm^2)	7.7			15.4			38.5			77		
$(t/0.1\ gm/cm^2)^{1/2}$	0.28			0.39			0.62			0.88		
$(\theta_1 / \theta_a)^2$	50.7			101			254			507		
b	3.79			4.46			5.39			6.10		
B	5.5			6.3			7.4			8.23		
θ_w / θ_1	2.07			2.26			2.49			2.65		
Elect. En. (Mev)	2	3	4	2	3	4	2	3	4	2	3	4
θ_1 (deg)	4.92	3.45	2.66	6.94	4.88	3.76	11.0	7.7	5.94	15.5	10.9	8.4
θ_w (deg)	10.2	7.13	5.51	15.7	11.0	8.52	27.4	19.2	14.8	41.0	28.3	22.2
% e^- within												
± 5°	48	66	80	27	44	57	13	21	30	6	12	18
± 8°	73	90	96	50	68	82	24	39	53	13	23	34
± 10°	85	95	97	62	80	92	32	51	64	19	30	46
± 15°	97	99	99	83	96	98	53	72	86	32	51	64
Relative Yield in												
± 5°	29	40	49	33	54	69	39	64	91	39	73	109
± 8°	45	55	59	61	83	100	73	118	161	79	140	207
± 10°	53	58	59	76	98	112	97	155	195	115	183	280
± 15°	59	60	60	102	117	120	161	219	262	195	310	390

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